Circular Fuzzy Iris Segmentation Using Isolines, Sine Law and Lookup Tables

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Abstract This paper proposes a new and original fuzzy approach to circular iris segmentation based on *isolines*, *sine law* and *lookup tables*. All isolines found in the eye image together form the space in which the inner and outer boundaries of the iris are searched for, while the sine law is used for identifying clusters of concyclic points within any given and possibly noisy isoline. The new segmentation procedure proved a failure rate of 5.83% when tested on 116`564 eye images (LG2200 subset of ND-CrossSenssor-Iris-2013 database) at an average speed of four images per second on a single Xeon L5420 core.

Introduction

One of the latest trends in iris segmentation is to use active contours for finding the inner and outer iris boundaries. Ultimately, Daugman has sparked our interest in this matter, after he announced he adopted the active contours as an instrument for iris segmentation [4].

The generic approach within all active contours techniques is to start with an initial curve and to evolve iteratively its shape and its position according to some conditions (under the pressure of a contour model, for example [6]) up to the point where some stopping criteria are satisfied (while hoping that the object of interest will be correctly marked by the final position and shape of the curve).

The issues concerning us in this context are the following:

- If the attempt to map a complex mathematical model on an image has the effect of writing something to the image, instead of just reading something from it;
- If the eye images (collections of discrete points) actually satisfy the ideal conditions in which contour stability and convergence [3] are guaranteed in models of continuous geometrical spaces;
- If the mathematical model mapped on the eye images is something intrinsic or extrinsic to the image (is discovered as belonging there naturally or is imposed);
- If all of these matters mentioned above translate into excessive computational costs with respect to the result.

Briefly, our approach is to consider each of the inner and the outer boundaries of the iris as being simultaneously an almost (fuzzy) circular isoline and a (fuzzy) *strong isoline edge*, where the notion of *isoline edge* is defined below. Circularity is tested using sine law and lookup tables (instead of using Hough transform, which is much computationally expensive). The sine law allows for identifying clusters of concyclic points within any given fuzzy (and possibly) noisy isoline, whereas the lookup tables are indexing sine values that follow to be used for an easy and fast computation of the radii and centers.

This paper continues by commenting some related articles (in the subsection that follows), by detailing the principles underlying the proposed segmentation approach (in the 2nd section), by presenting the proposed segmentation procedure, the experimental results and our conclusions.

Related Works

Efficiently finding the iris boundaries using energy/force based active contours (known as snakes, [4]) strongly depends on initializing the search algorithm with a particular snake position found pretty close to the final/desired one. It happens this way because the theoretical model of these active contours is not very well supported by the eye images, and therefore, the errors accumulate in direct proportion with the number of iterations in such a manner, that sometimes, the target boundary can even be traversed by the iterating snake. Besides, this active contour model does not support merging and splitting (however, the extended snake model presented in [6] overcomes this issue). In general, the snakes are considered elastic curves continuously deformable under the action of some user defined *image forces* towards minimization of some user defined energy field. The so called *image forces* are responsible for snake *self-cohesion*, *smoothness*, *expansion* and *contraction*.

The idea of level-set based active contours is to express the current position as a constant level set of a potential function which slightly changes over time carrying/pushing the active contour to its next position [6], [7], [16]. By design, this active contour model actually supports splitting and merging, but at the cost of a greater computational effort.

Geodesic active contours got their names mainly by analogy with some particular situations in geometry and mechanics, their purpose being to extract object boundaries as a minimal length geodesic. These things go back to Maupertuis' Principle that tells [2] in which conditions a snake is a geodesic, i.e when a given solution minimizing an energy in an Euclidian (flat) space can be expressed as a solution minimizing a distance in a Riemannian (curved and twisted) space. However, hidden behind this principle there is a belief (or an assumption) that any physical space is metrizable in the same manner in all of its points (there is an intrinsic and global operator defining the Riemannian distance, in the same way within the entire image) - an assumption that is generally false (an example will be given below when our approach to this problem will be discussed on eye images). When it comes for estimating computational complexity of such technique in relation with the outcome, one could see (for example in Fig. 13.g from [14] and in Fig. 5 from [13]) that the number of iterations necessary for finding a good enough approximate of the outer iris boundary can vary (1800 vs. 600 iterations). In the same figures, it can be seen that the intermediate positions the snake is passing through are quite meaningless in human understanding. By iterating in this manner, one writes something to the eye image instead reading suitable candidates that are actually there in the image, until finding the target boundary, which is also there.

Principles underlying the proposed segmentation approach

We would like to say it was very simple to find this new iris segmentation procedure in a lucky our, day, week or even month, but this is very far from being true. A lot of things that we read or we experienced during the last 9 years guided us in the current point of our research. During this entire period, while revisiting a number of computing disciplines, we searched for answers on how to define efficient iris segmentation procedures with less computational costs for the same or better results. It happen that these answers came to us not specifically for the purpose of iris segmentation, but for an entire class of searching problems as a collection of notions and principles useful each time when someone feels or knows that the speed-accuracy balance is not the right one. Two of them (in our view, the most important after those of *imperfect* / *consistent experimental frameworks* [9] and *Consistent Biometry* [10]) are the notion of *vital space* and the *principle of deflation* - another two essential bricks completing the formal apparatus initially developed in [9] and [10], mainly around essential logical and methodological matters within biometrics. Regardless its title, this entire section is about practicing iris segmentation efficiently, not about theoretical principles.

The global vital space of a grayscale image

In order to maintain the state-space of our algorithm inside a minimal volume of intrinsic image properties, an eye image (as much as any other two dimensional grayscale image) is viewed here as a punctual distribution of chromatic information (quantized on a discrete scale) spread in a rectangular domain of discrete pixels. Hence the natural global frame coordinate system for the eye image is three-dimensional, discrete, and spans a volume of a discrete parallelepiped (built with cubic pixels) having a discrete rectangle of pixels as the horizontal base, and the chromatic discrete scale in any point of the base oriented in the normal upward direction defined such that, together, the column (x), row (y) and chromatic (z) axes to form a right-handed coordinate system. This volume, further referred to as the *vital space* of a grayscale image, is a granule of space having an extended convexity property: for any two points found inside it, the shortest paths in between them and the shortest paths from the two points to all coordinate axes and planes (where distance is defined by taxi-cab norm) are all entirely found inside it, also. In other words, any pixel of the image is reachable from origin and from any other point of its *vital space* using paths within this space. Moreover, given a set of three points A, B, C, inside this space, any two of them are connected by a lot of paths passing through the third one and having minimal length (in taxi-cab norm). In a formal view, we ask if it is possible that all intrinsic properties of the image to be expressed in a closed formal language using constants, variables, operators, predicates and functions defined solely inside its *vital space*. Regardless the answer to this ambitious question, it is evident that any iterative procedure able to return a handler to an object of interest within the image has to traverse the vital space boundary if it starts somewhere outside this space. Hence, in order to obtain fast processing algorithms, it is preferably to start them within the vital space or, at most, on vital space boundary instead of starting them outside and passing through the boundary towards the object of interest.

Search space deflation

As a graph, the *vital space* is fully connected. Therefore, the ensemble (R, V_s, P, I) formed with the right-handed referential - R, the *vital space* V_s, the set of all paths within it – P, and the 8-bit eye image I, is an implicit over-determined description of the image that can be exhausted completely in exponential time $2^{255*r*c}$ (where *r* and *c* are the number of rows and columns of the image) as upper bound. In this context, deflation principle is a principle of action, stating that any search space must be decimated first, as much as possible, by any means, in order to decrease computational and time complexity of the search process.

When searching for a given object of interest within an image, deflation process can occur in successive stages, each of them corresponding to a one step genus-to-species reduction in which the complement of a species is decimated from the search space. If a formal, logical and hierarchically nested ontological model object-species-genus exists for the object of interest, deflation should be easy. Unfortunately, the object of interest is often defined in natural language, not in data and a programming language - this is the main source of troubles.

When the definition of the target object is wrong or not rigorous enough, more objects (belonging actually in other species) or fewer objects of interest are found, error situations known as False Accept and False Reject cases. This is why our efforts were directed to finding suitable computational definitions for the inner and outer boundaries of the iris in the first place, such that, the process of finding them to include abrupt efficient deflation stages for the initial search and vital spaces.

Defining a local vital space for a grayscale image using isolines

Each time when this is possible (with reasonable computational effort), we would like to know a local coordinate system that moves across the image. In this view, a kind of tangent space is mapped onto the image (an analogy with the tangent space across a manifold).

First important characteristic of this tangent space is that it is not known (defined) in all pixels of the image (a first deflation step, and a first fuzzification step – the information becomes partial), whereas the second is that it does not preserve its orientation for all pixels where it is defined, nor the angles in between its axes. We are not bound to the idea of imposing an orthogonal local coordinate system (or other classical referential) just because it would be easier for us to work with. On the contrary, we are curious to find what is there, what is supported there by the image itself.

A third important characteristic of our approach is that we consider (numerical) differentiability (implemented by finite differences) a matter of degree (simply because its geometrical appearance, namely the smoothness, is also a matter o degree). Hence, an image cannot be modeled here as a differentiable manifold. On the contrary, (numerical) lateral partial derivatives will be very often far from matching each other.

Another important aspect, concerning the degeneracy of coordinate curves, will be commented and illustrated by example below, while talking about isolines.

The particular way in which an intrinsic local coordinate system varies across the eye image is itself a feature that can be used in detecting regions and/or objects of interest.

Fuzzy isolines

Ideally, a crisp isoline within an eye image is a crisp curve whose points share the same chromatic level. Since the existence of this ideal form of isoline is not guaranteed, we must weaken this concept up to *fuzzy isoline* (f-isoline), by specifying a way of interpreting small chromatic variations as being fuzzy nil, so that, a fuzzy isoline is now a curve along which the chromatic variation is fuzzy nil. In other words, all points within an *f-isoline* are fuzzy critical points of the fuzzy chromatic gradient along the *f-isoline*. Semantically speaking, the concept of fuzzy isoline is a handler pointing to curves with minimal (*fuzzy nil*) chromatic variability. The fuzzy aspect across the *f-isoline* (incertitude in exact line-column position) is neglected (prior to circle finding step of our approach) in favor of the fuzzy aspect along the *f-isoline* (incertitude in exact chromatic value). Hence, it is easy to imagine the *f-geometry* [15] of a circular *f-isoline* as if we spray a straight horizontal line on a paper and then use that paper as a lateral surface for a cylinder. If the incertitude across a circular isoline is not neglected, its *f-geometry* becomes a sprayed torus. In both cases, the prototype curve [15] of the both *f-isolines* is a circle. In what follows here, in order to avoid complex graphical representation, the *f-isolines* will be graphically represented by their crisp prototype counterparts, as in Figure 1.b. The *f-isolines* form the first family of local coordinate curves and span local directions of fuzzy nil (minimal) chromatic variation.

Degenerated f-isolines

A closed *f-isoline* is a cycle within the image vital space regarded as a graph. If the interior of a closed *f-isoline* is large enough and is not containing another isoline, that *f-isoline* is the border of a region with fuzzy constant chromatic value (a fuzzy *isosurface*). Hence, a fuzzy isoline can degenerate to a fuzzy isosurface, meaning that the local passage of fuzzy nil chromatic variation is not narrow, but wide. In such image regions, it is impossible to define the second family of local

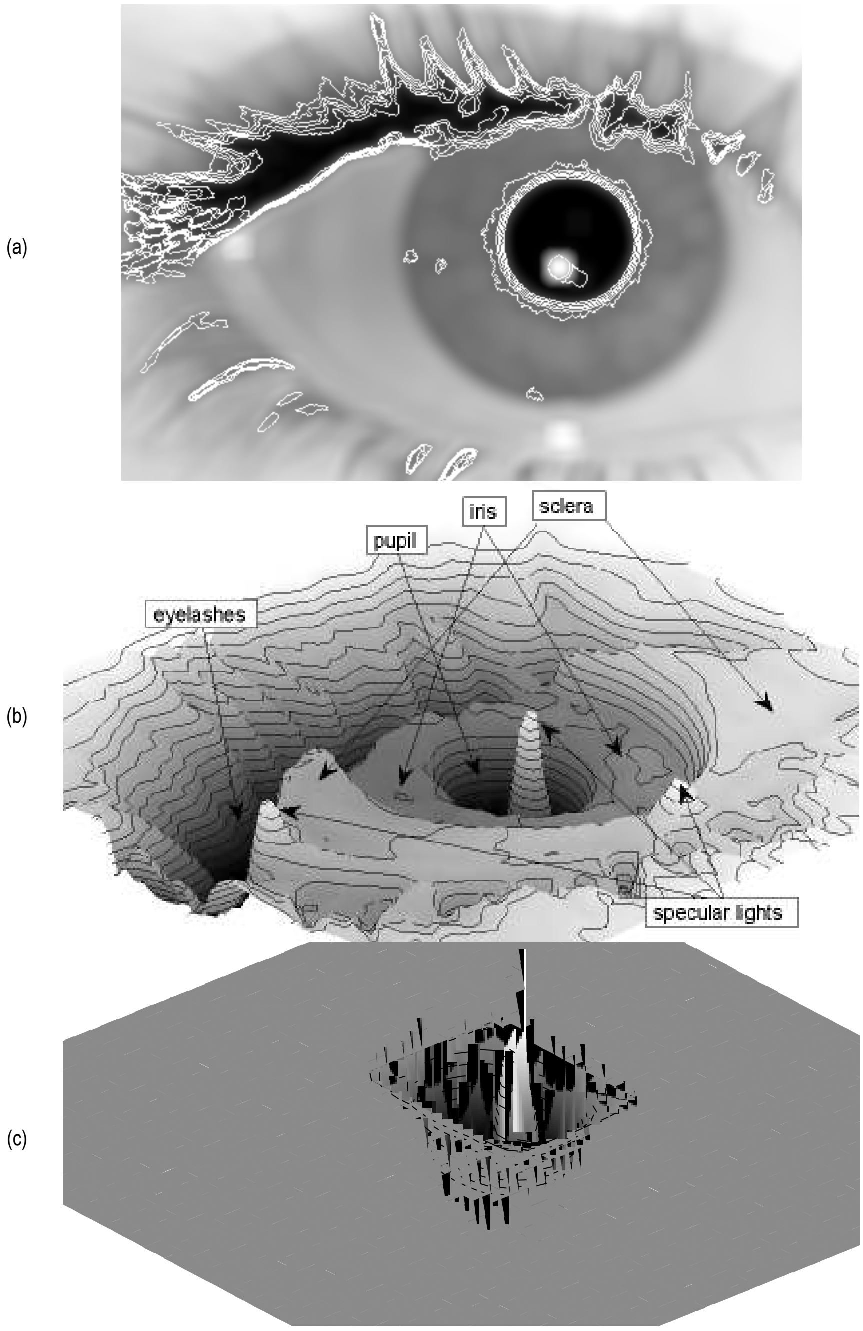

Fig. 1. Detecting the pupil: **(a)** – Fuzzy isolines within the lower chromatic of the eye image; **(b)** - The f-isolines within an eye image represented by their crisp prototype counterparts on 32 chromatic levels; **(c)** – Pupil isoline chain

coordinate curves along which chromatic variation is maximal, simply because there, the chromatic variation is locally fuzzy nil in all directions. The possibility of defining directions of maximal chromatic variation exists only for pixels situated on the border of the degenerated closed *f-isoline*, cases in which, the directions of maximal chromatic variation are all pointing out or in, across the isoline. When the interior of a degenerate closed *f-isoline* has fuzzy nil or small area, it is called *a fuzzy spot*. A fuzzy membership function can be defined for identifying white, gray and black spots, as well as white, gray and black isosurfaces.

Essential local chromatic variations

Two kinds of *local directions* are considered essential here: those of *minimal* and *maximal* chromatic variation, respectively. Let us consider the set of all pixels covered by isolines (IP - a pretty sparse truncation of the eye image vital space). For all of them, local directions of fuzzy nil (minimal) chromatic variation are known (along the isolines that are going through there). For only a part of them, directions of maximal chromatic variation can be defined also, while using solely the pixels within IP, as follows below:

-The *interface* I between two isolines is defined here as the set of all sliding segments drawn from one isoline to the other and having the property that they are not intersecting any other third isoline in between the two given isolines, nor the isolines on which they slides with their end points. Further, we will say that two isolines *interface each other* in point P (which is on one of them, or equivalently, that the two isolines are *locally adjacent in* P, or *locally connected in* P) if there is a non-empty interface in between them containing a segment starting in or ending with P. An isoline is interfaced on one side in one of its points P, if there is another isoline interfacing it on that side in P (i.e. degenerated closed isolines are interfaced on at most one side).

-An isoline is *captive* on a side if it is interfaced on that side in all of its points by other one and only isoline. The restriction of the captivity relation to the set of closed isolines is symmetric.

-For given *crisp prototype* [15] isolines and their fuzzy counterparts [15], the captivity relation is weakened here up to the point where we say that a fuzzy isoline is *f-captive* on a side if it is interfaced on that side in *almost all of its points* by other one and only closed fuzzy isoline.

-An *isochain* (a fuzzy isochain) is defined here as any sequence containing at least three (fuzzy) isolines that are pair-wise (fuzzy) captive to each other. The isolines within such a sequence will be further referred to as being (fuzzy) *globally chained*.

-Two isolines are said to be *connected globally* if they are captive to each other. An intensive parameter '*cs*' expressing the *connection strength* as a matter of degree is introduced here as the percentile of all points in which the two isolines are interfaced. For all pairs of successive (pair-wise captive) isolines found within an isochain, connection strength parameter *cs* is *unitary* and *maximal*.

-Two isolines are said to be *connected locally* if there is a common neighborhood such that their restrictions inside there appear as being connected globally.

-An isoline edge is defined now as any isoline ending an isochain or one/those supporting the maximum local chromatic variation along an isochain. Its fuzzy counterpart is defined by analogy.

-Given a point P on an isoline IL_1 and a second isoline IL_2 interfacing IL_1 on one side in P (where the two isolines can share either different or the same chromatic level), *a local direction of maximal variation* in P is the vector defined by the shortest path trough P found within the interface of IL_1 and IL_2. If the shortest path from P to IL_2 is not unique, *a local direction of maximal variation* in P is the vector PP' where P' is situated on IL2 and PP' is matching the direction of the resultant of all shortest paths from P to IL_2.

-Given a point P on an isoline IL and two other isolines IL_1 and IL_2 interfacing IL on the same side in P, then, regardless the chromatic levels of the isolines, in general there are two different local directions of *essential variation* in P across IL, one maximal with respect to IL_1 and the other one maximal with respect to IL_2.

A local, isoline-based, vital space of the grayscale image

If a non-degenerated isoline IL has only two adjacent isolines IL_1, and IL_2, one on each of its sides, in a given point P, a local intrinsic coordinate system in P is given by two local directions of fuzzy nil chromatic variation (from P, back and forth along IL) and two local directions of maximal variation (across IL in P, one on each of its sides in P, towards IL_1 and IL_2, respectively).

Finally, a local intrinsic coordinate system exists for any pixel P found on an isoline IL and contains 2 local directions of fuzzy nil chromatic variation (back and forth from P, along the isoline) and N local directions of maximal chromatic variation (where N is the number of distinct isolines locally adjacent with IL in P). Hence, *the first family of local coordinate curves contains isolines, whereas the second family of local coordinate curves contains polygonal lines stepping in between locally adjacent isolines while simultaneously minimizing the distance and maximizing chromatic variation*.

The reunion of all vital spaces associated point-wise to all pixels covered by isolines is *the local isoline-based vital space* of the grayscale image. Keeping the computation inside this space (while searching for objects of interest) makes sense from two points of view: this space is the result of minimizing the volume while maximizing the information about the local chromatic variations, and secondly, this space is assumed practically *as it is*, not ideally – as having theoretic properties in terms of continuity and smoothness.

Along a closed isoline that is captive on both sides, the local intrinsic coordinate system reflecting the *essential* local chromatic variations in the current point P is 4-dimenssional containing *two* local directions of fuzzy nil chromatic variation (from P back and forth, along the isoline) and *two* directions of maximal chromatic variation across the isoline, towards the two isolines that captured it in between them.

Exhausting the *local isoline-based vital space* of a grayscale image is a way of evidencing how to traverse that image by stepping only on and in between (locally or globally) connected / adjacent isolines.

Low complexity circularity test based on sine law and lookup tables

Using the terminology defined above, from a practical point of view expressed in terms of intrinsic properties of the eye image, the pupil boundary (found within a standard eye image collected for biometrics purposes) is simultaneously: (1) *a fuzzy isoline*, (2) *fuzzy circular*, (3) *a member of an isochain* and (4) *a fuzzy isoline edge*. From all these four features, three were commented above, whereas the circularity is usually tested by applying *Hough Transform* on the edge matrix. Firstly, we are no longer considering edges, but *isolines*, *isochains* and *edge isolines* and their fuzzy counterparts. Secondly, we searched for an alternative to Hough voting (which is well known as being computationally expensive) and we found one way to estimate circularity and circular/concyclic clusters within the isolines using the sine law, as follows. Let us consider :

- an isoline IL,
- R - the set of rectangles defined by points within IL such that they have a vertical side,
- T – the set of triangles △ABC defined by points within IL, such that they have acute angles only, and either a horizontal AB or a vertical AB side, as illustrated in Figure 2.
- the circumcircle of △ABC heaving the radius **r**;
- c – the length of AB, H∈(AB) such that CH ⊥ AB;
- d – the distance from A to H,
- h – the length of CH;
- LT – a 3D lookup table containing $c/2\sin(\hat{C})$ values indexed after c, d, and h, according to sine law:

$$\mathrm{r}(\mathrm{c},\mathrm{d},\mathrm{h}) = \frac{\mathrm{c}}{2\sin\left(\widehat{\mathrm{C}(\mathrm{c},\mathrm{d},\mathrm{h})}\right)} \quad (1)$$

- a fuzzy predictor p_r, of low computational complexity (linear with respect to the cardinal of T), defined on T and selecting the most voted radius over the set T, radius that corresponds to the maximum point within a histogram of a signal formed by reading (not by computing) radius values corresponding to unique (c, d, h) indexing triplets in the lookup table LT;
- a fuzzy predictor p_c, of low computational complexity (linear with respect to the cardinal of R), defined on R and selecting the most voted center over the set R. Voting mechanism is based on the fact that any rectangle in R votes its center as being the center of a concyclic cluster found within the isoline. These centers are found with maximal efficiency, by averaging two abscise values and two ordinate values as in Figure 3 (not by computing square roots for some sums of squares).

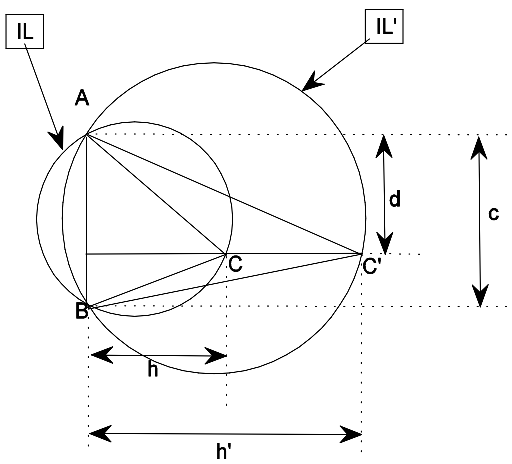

Fig.2. The set T and the indexing scheme within the lookup table LT for $r = c/2\sin(\hat{C})$ values corresponding to unique (c, d, h) triplets - see also formula (1).

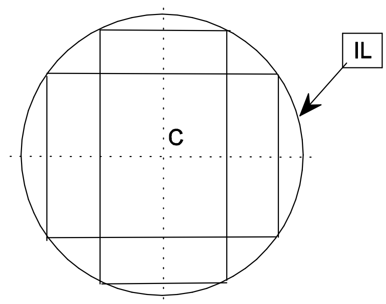

Fig. 3. For any circular isoline, by sharing the same vertical and the same horizontal symmetry axes, all concyclic rectangles within the set R vote for the same center, whose coordinates are computed by averaging two column and two line indices, respectively.

Advantages

By design, the two fuzzy predictors p_c and p_r from above, both do their job well, even when the isoline is not circular on its entire length: a specular light or other occlusion situated near the inner iris boundary usually modifies locally the isoline path, but this perturbation cannot happen simultaneously in all corners of the rectangles within R, or in all corners of the triangles within T.

The lookup table LT is computed only once and used over and over again as a replacement for its much computationally expensive alternative, namely Hough accumulator. From the point of view of a system designer, this means that the entire computational effort of writing this table on either a ROM (read only memory) or a RAM can be done once, prior to exploitation (during system calibration) or during system boot, respectively.

Iris segmentation procedure based on isolines, sine law and lookup tables

With the terminology introduced in the previous sections, pupil boundary is an *almost* circular *edge isoline* whose radius r_p belongs in a given range rr_p, isoline belonging in a *highly connected chain* (almost unitary connection strength between its adjacent isolines) that makes the *longest passage* to one of the *darkest* and *almost circular* isoline found outside a given image border. Any *isoline chain* that candidates for the inner iris boundary is discarded if no left and right outer boundaries are found around it. Shortly, the *candidate pupil chains* are discarded by backtracking after successful outer left/right iris boundaries detection. Each of the latter also belongs in a *chain* (possibly a *weakly connected chain*) that makes the passage between two adaptively determined chromatic thresholds: one lower threshold (*lt*) detected as the chromatic value of the whitest isoline of the current *pupil chain candidate* and one upper threshold (*ut*) identified as the first regional (i.e. non-spot) maxima found across the iris radius range on the left and right sides of candidate pupil chain (hopefully on the sclera, but not mandatory).

Despite, theoretically, it could be possible to finalize the search for the iris boundaries with two or more candidates, this never happen practically on LG2200 database, and therefore, we haven't established a procedure for selecting the most suitable candidate triplet (inner circular boundary, left outer circular boundary, right outer circular boundary) in this case.

Over all, pupil and iris are considered concentric, whereas the outer circular iris boundary is defined by the smallest radius detected for the left and right outer circular boundaries. This strategy was first used in [12] and is motivated by the fact that doing that is not giving guarantees on the accuracy of segmenting each individual eye image, but guarantees that the iris regions neglected in any two different standard eye images taken from the same user are almost the same (bigger the eccentricity of the iris, wider the area of neglected iris region).

The pseudo-code for the proposed segmentation procedure is as follows:

1. INPUTS:
   - 1.1 the current standard infra-red eye image **EI**;
   - 1.2 the given pupil radius range and its extreme values: $\mathbf{rr_p}$;= [$\mathbf{mrr_p}$, $\mathbf{Mrr_p}$];
   - 1.3 the given iris radius range and its extreme values: $\mathbf{rr_i}$; = [$\mathbf{mrr_i}$, $\mathbf{Mrr_i}$];
   - 1.4 the width of the image border **ib**;
2. Expand the local vital space of the eye image as a collection of local directions of minimal & maximal chromatic variations (along & across the isolines found in the lower image chromatic).
3. Identify the set of pupil candidates **PC** as containing the first four longest and most connected chains fitted inside $\mathbf{2Mrr_p}$ x $\mathbf{2Mrr_p}$ squares and formed with almost circular isolines whose chromatic values are descending to black (Figure 1.c);
4. For each pupil candidate **pc** in **PC**,
   4.1 Get the lower chromatic threshold ***lt*** detected as the chromatic value of the whitest

outer isoline belonging in the current *pupil chain candidate*;

4.2 Get the upper chromatic thresholds ***utl*** and ***utr*** identified as the first regional (i.e. non-spot) maxima found across the iris radius range on the left and right sides of candidate pupil chain;

4.3 Infer pupil radius $\mathbf{r_p}$ (Figure 2) and pupil center position $\mathbf{c_p}$ (Figure 3)

4.4 Get BN be a \`butterfly\` neighborhood whose wings extend from the left and from the right of the pupil candidate, for 60° around the horizontal of pupil center (±30° above/under).

4.5 To the left and to the right of BN, find left and rigt outer iris boundary candidates (**libc** and **ribc**) as the longest and most connected chains making the passage from ***lt*** to ***utl*** and to ***utr***, respectively.

4.6 Memorize the current (**pc, libc, ribc**) triplet candidate formed with the edge isolines within the three corresponding chain candidates in the set of possible solutions **PS**.

5. If PS is empty,

5.1 report that the current image does not contain a detectable iris.

Else,

5.2 for the best candidate in **PS**:

5.2.1 Infer iris radius $\mathbf{r_i}$ as the minimum between the radii of **libc** and **ribc**;

5.2.2 Mark pupil circle, and both iris circles (corresponding to **libc** and **ribc**);

5.2.3 Get BN be a \`butterfly\` neighborhood whose wings extend from the top and to the bottom of the pupil candidate, for 60° around the vertical of the pupil center (±30° right/left), find the edge isolines that define top/bottom eye occlusions (if any) and mark top/bottom eye occlusions (if any) by filling the regions found inbetween the outer iris boundary and top/bottom edge isolines previously determined.

6. OUTPUT: either

6.1 an empty vector (no iris found)

or

6.2 two occlusion masks and a vector containing two center coordinates, the inner and outer radii of the iris.

Experimental Results

The new segmentation procedure proposed here proved a failure rate of 5.83% when tested on 116\`564 eye images (LG2200 subset of ND-CrossSenssor-Iris-2013 database, [8]) at an average speed of four images per second on a single Xeon L5420 core, where a segmentation failure means that no suitable candidates for inner and outer iris boundaries have been found within an image. However, we know from our previous work on this ‘difficult’ database that almost a half of the situation in which segmentation fails (around 3000 eye images), it does that because those images are improper with respect to iris recognition. Such examples are illustrated here in Fig. 5: for some of them there is no pupil chain candidate to be found because there is no circular pupil present; in other cases, a circular pupil is present but it is not black enough, or position of the iris is to close to image border.

The cases of successful segmentation are illustrated below in Fig. 4 where one can see examples for: horizontal / vertical butterfly neighborhoods of the iris center (see 4.4 and 5.2.3 in the pseudo-code of the proposed segmentation procedure), left and rigt outer iris boundary candidates represented as the two circles (**libc**/**ribc** – see the instructions 5.2.1, 5.2.2 in the pseudo-code above) and the upper occlusion mask induced by the presence of eyelashes over the iris area.

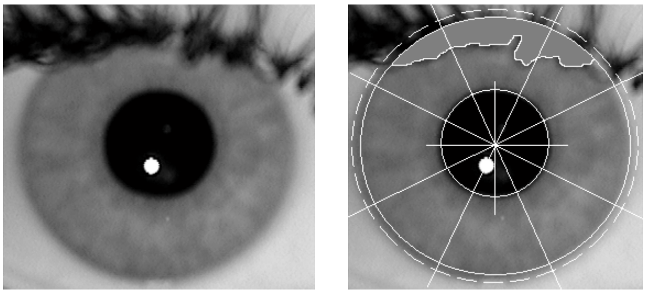

Fig. 4. Iris segmentation based on isolines, sine law and lookup tables

Fig. 5. Eye images that are not able to deliver at least one suitable triplet candidate (**pc, libc, ribc**) for pupil (inner), left and right (outer) iris boundaries - see above, the steps (4.6) and (5) in the pseudo-code of the proposed segmentation procedure.

Conclusions

This paper proposed a new and original fuzzy approach to circular iris segmentation based on *isolines*, *sine law* and *lookup tables*. All isolines found in the eye image together form the space in which the inner and outer boundaries of the iris are searched for, while the sine law was used for identifying clusters of concyclic points within any given and possibly noisy isoline.

The main difference between the related works and our approach is that we only read something from the eye image searching for three isoline chains matching certain conditions that together qualify them as being inner and left/right outer boundaries of the iris.

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